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Perfect State Transfer and Entanglement Generation with Coupled Cavities

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Abstract We study atomic state transfer and entanglement generation when the N atomic ensemble is trapped in two coupled cavities. We show that based on the collective interaction between the atoms and local cavity fields an ideal quantum state transfer can be realized if some special conditions are satisfied. In addition, the maximal atom entangled state can be achieved. The effect of the cavity losses on the quantum processes is also studied.

Keywords Quantum state transfer · Atomic entanglement

1 Introduction

The generation of entanglement as well as the transfer of a quantum state between distant systems have been significant in the field of quantum information science for distributing and processing information [1]. Cavity quantum-electrodynamics (QED) systems can provide mechanism for communicating between high-Q cavities because of its important paradigm for coherent coupling of atomic and photon qubits [2]. As is well known, atoms are particularly well suited for storing qubits in their long-lived internal states, while photons are the best qubit carriers for fast and reliable communication over long distances. Based on cavity QED systems, several schemes have been proposed to realize quantum state transfer or engineer entanglement between atoms trapped in two distant optical cavities [3–7]. Through a coherent coupling mediated by an optical fiber, Mancini et al. [5] proposed a scheme to generate an effective interaction and entanglement between two atoms placed in distant cavities. Yin et al. [6] investigated the quantum state transfer between multiple two-level atoms trapped in single-mode cavities spatially separated and connected by an optical fiber. Recently, a scheme is presented to realize quantum state transfer between distant nodes of a quantum network via adiabatic passage [7]. Most recently, intense interest has arisen for an array of coupled cavities because spin chain [8, 9], Bose–Hubbard interaction and

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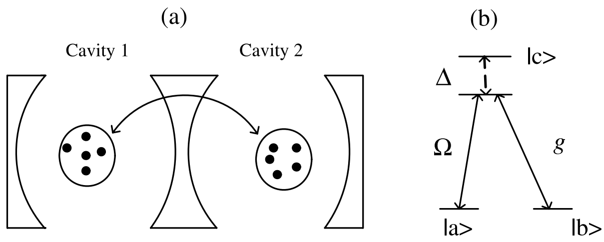

Fig. 1 (**a**) N three-level atoms trapped in two coupled cavities. The illustration of experiment setup. (**b**) The configuration of the atomic level

superfluid-to-Mott-insulator quantum phase transition can be simulated in these structures [10–12]. Cho et al. [13] introduced a method for generating two-photon, two-atom, or atom-photon entangled states with a two-coupled cavities each having a single atom. Ogden [14] investigated the dynamics of the same system with [13] over a wide range of detuning and hopping values.

In experiment, it is more practical that multiple atoms are trapped in each of cavities than single atom does, and various decoherence processes such as spontaneous emission of atoms and photon leakage out of cavities are inevitable. In the present study, we consider a scheme similar to that [13, 14]. However, we consider multiple three-level atoms in stead of a single atom. In order to avoid atomic spontaneous emission, we employ three-level Ramman process equal to two-level atom. The decays of the cavities are included. We show that an ideal quantum state transfer can be obtained and the maximum atomic entanglement can be achieved when the atoms collectively interact with the local cavity fields. The cavities leakage is also included.

The paper is structured as follows: In Sect. 2, we present the model system under consideration. In Sect. 3, the quantum state transfer is generated and we also calculate the probability of the quantum state transfer. In Sect. 4, the generation of entangled state is provided and discussed. In Sect. 5, the influence of cavity losses is investigated. Finally, we conclude with a brief summary in Sect. 6.

2 Model and the Hamiltonian

The system under consideration consists of multiple three-level atoms trapped in two directed coupled optical cavities (see Fig. 1(a)). The atoms interact with the local cavity fields. The size of the space occupied by the atoms is assumed to be far smaller than the wavelength of the cavity field so all the atoms in each cavity could feel the same field. Meanwhile, we also assume that the atoms are so separated that they do not have direct interaction each other. As shown in Fig. 1(b), the atoms have the same level structure of a three-state system with the excited state $|c_i\rangle$, and two ground states $|a_i\rangle$, $|b_i\rangle$. The cavity mode with coupling constant g_j interacts with the transition $|c_i\rangle \leftrightarrow |b_i\rangle$, while the laser field with Rabi frequency Ω_j couples to the transition $|c_i\rangle \leftrightarrow |a_i\rangle$. Δ is the frequency detuning. Considering the rotating wave approximation [15], the interaction Hamiltonian of the atom-field system in the interaction picture can be written as ($\hbar = 1$)

$$H_j(t) = \sum_{i=1}^{N} \Omega (e^{-i\Delta t}|a_i\rangle\langle c_i| + e^{i\Delta t}|c_i\rangle\langle a_i|) + g(a_j^{\dagger} e^{-i\Delta t}|b_i\rangle\langle c_i| + a_j e^{i\Delta t}|c_i\rangle\langle b_i|) \quad (j = 1, 2), \tag{1}$$

where $a_j^\dagger$, a_j are the creation and annihilation operators for photons in the cavity j ($j = 1, 2$), respectively. N is the amount of atoms in each cavity. We suppose that g and Ω are all the same for any atoms and both are real numbers in this paper. By applying standard quantum optical techniques [16], under the large-detuning condition, $\Delta \gg \{\Omega, g\}$, we can adiabatically eliminate the excited state and obtain the effective Hamiltonian

$$H_j^{eff} = \sum_{i=1}^{N} -\frac{\Omega g}{\Delta}(a_j|a_i\rangle\langle b_i| + a_j^\dagger|b_i\rangle\langle a_i|). \tag{2}$$

The raising and lowering operators for the atoms are defined as

$$S_j^\pm = \sum_{i=1}^{N} \sigma_i^\pm(j), \tag{3}$$

where $\sigma_i^+(j) = |a_i\rangle_j\langle b_i|, \sigma_i^-(j) = (\sigma_i^+)^\dagger$ are the corresponding spin-flip operators up and down, respectively. So, we can rewrite the effective Hamiltonian as

$$H_j^{eff} = \lambda S_j^+ a_j + H.c., \tag{4}$$

where $\lambda = -\frac{\Omega g}{\Delta}$ is the effective coupling constant between the cavity j and the atoms.

Next step, we consider that two cavities are coupled directly. In this case, hopping between the cavity fields is achieved by the overlap of evanescent fields out of the intermediate cavity mirror. Then the interaction Hamiltonian for cavity photons can be simply written as [10]

$$H_I = J(a_2^\dagger a_1 + a_1^\dagger a_2), \tag{5}$$

where J is the rate of intercavity hopping of photons. The Hamiltonian of the total atom-cavity system takes a simple form

$$H_{tot} = H_1^{eff} + H_2^{eff} + H_I. \tag{6}$$

In the following part, we will use the above Hamiltonian to calculate quantum state transfer and atomic entanglement generation.

3 Quantum State Transfer

In our scheme, N three-level atoms are trapped in each of the cavities. We define the state $|0, b\rangle = \sum_{i=1}^{N} |b_i\rangle$ which means all of the N atoms are in the ground state $|b\rangle$ and $|1, a\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |\ldots a_i \ldots\rangle$ which means $N - 1$ atoms are in the ground state $|b\rangle$, and one atom is in the excited state $|a\rangle$. We have $S^+|0, b\rangle = \sqrt{N}|1, a\rangle$, $S^-|1, a\rangle = \sqrt{N}|0, b\rangle$. We assume that at the initial time all the atoms in the first cavity are in a superposition state $\alpha|0, b\rangle_1 + \beta|1, a\rangle_1$, here α and β satisfy the condition $|\alpha|^2 + |\beta|^2 = 1$. And the atoms in the second cavity are in the state $|0, b\rangle_2$. Suppose that at the initial time both of the cavities are in the vacuum state $|00\rangle$. So the initial state of the whole system $|\Psi(0)\rangle$ is the superposition state $(\alpha|0, b\rangle_1 + \beta|1, a\rangle_1) \otimes |0, b\rangle_2 \otimes |00\rangle_f$. We aim to transfer the information carried in atoms of the first cavity to the atoms of the second cavity. So, the goal quantum state should fulfill the operation

$$(\alpha|0,b\rangle_1+\beta|1,a\rangle_1)\otimes|0,b\rangle_2\to|0,b\rangle_1\otimes(\alpha|0,b\rangle_2+\beta|1,a\rangle_2). \tag{7}$$

We notice that the part of the initial state $|0,b\rangle_1|0,b\rangle_2$ does not evolve. Thus the other part $|1,a\rangle_1|0,b\rangle_2$ will evolve in the domination of the Schrödinger equation ($\hbar=1$)

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle=\hat{H}|\Psi(t)\rangle, \tag{8}$$

where $|\Psi(t)\rangle$ denotes the state of system at time t, H is given by formula (6). We express

$$|\Psi(t)\rangle=\sum_{i=1}^{4}C_i|\Phi_i\rangle, \tag{9}$$

where $|\Phi_i\rangle$ are defined as

$$\begin{aligned}|\Phi_1\rangle&=|1,a\rangle_1|0,b\rangle_2|00\rangle_f\\|\Phi_2\rangle&=|0,b\rangle_1|1,a\rangle_2|00\rangle_f\\|\Phi_3\rangle&=|0,b\rangle_1|0,b\rangle_2|10\rangle_f\\|\Phi_4\rangle&=|0,b\rangle_1|0,b\rangle_2|01\rangle_f.\end{aligned} \tag{10}$$

Associating with the (8), (9) and (10), we can get the expressions of coefficient C_i,

$$\begin{aligned}C_1&=\cos\frac{Jt}{2}\cos\frac{at}{2}+\frac{J}{a}\sin\frac{Jt}{2}\sin\frac{at}{2}\\C_2&=-i\sin\frac{Jt}{2}\cos\frac{at}{2}+i\frac{J}{a}\sin\frac{at}{2}\cos\frac{Jt}{2}\\C_3&=-\frac{2\lambda\sqrt{N}}{a}i\sin\frac{at}{2}\cos\frac{Jt}{2}\\C_4&=-\frac{2\lambda\sqrt{N}}{a}\sin\frac{at}{2}\sin\frac{Jt}{2},\end{aligned} \tag{11}$$

where $a=\sqrt{4N\lambda^2+J^2}$. It is easily shown that when $Jt=k\pi, k=1,3,5,\ldots,$ $\frac{at}{2}=k'\pi, k'=1,2,3,\ldots,$ the state $|1,a\rangle_1\otimes|0,b\rangle_2\otimes|00\rangle_f$ evolves into the state $|0,b\rangle_1\otimes|1,a\rangle_2\otimes|00\rangle_f$. Combining the above results, we implement the perfect transfer $(\alpha|0,b\rangle_1+\beta|1,a\rangle_1)\otimes|0,b\rangle_2\to|0,b\rangle_1\otimes(\alpha|0,b\rangle_2+\beta|1,a\rangle_2)$. The success probability of the state transfer can be calculated as

$$\begin{aligned}F&=|C_2|^2\\&=\left(\sin\frac{Jt}{2}\cos\frac{at}{2}\right)^2-2\frac{J}{a}\sin\frac{Jt}{2}\cos\frac{Jt}{2}\sin\frac{at}{2}\\&\quad\times\cos\frac{at}{2}+\left(\frac{J}{a}\right)^2\left(\sin\frac{at}{2}\cos\frac{Jt}{2}\right)^2.\end{aligned} \tag{12}$$

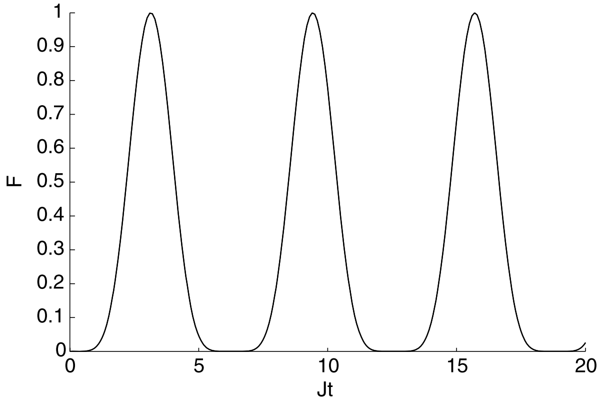

Fig. 2 The probability amplitude of the quantum state transfer as a function of Jt. $g_1 = g_2 = 1$, $\Omega_1 = \Omega_2 = 2$, $\Delta_1 = \Delta_2 = 20$, $N = 75$

In Fig. 2, the probability of the state transfer is plotted as a function of Jt. We find that under this group of parameters, the probability amplitude periodically reaches the maximal value 1. From the (11), it is easy to see that if the two conditions $Jt = k\pi$, $k = 1, 3, 5, \ldots$, $\frac{at}{2} = k'\pi$, $k' = 1, 2, 3, \ldots$ are intensely fulfilled, the value of probability amplitude is always 1, which means we realize a full quantum state transfer. When the value is 0, we fail to realize the quantum state transfer.

4 The Generation of Entangled State

In this section, we will draw attention to how to generate the entangled state of the atoms and fields, respectively. We still consider the evolution of initial state $|1, a\rangle_1 |0, b\rangle_2$, expressed in (9) and (10). At first, we trace over the part of the field and obtain the density matrix of the atoms ρ_a, which has the form as follows

$$\rho_a = \mathrm{Tr}_f |\Psi(t)\rangle\langle\Psi(t)| = \begin{pmatrix} |C_3|^2 + |C_4|^2 & 0 & 0 & 0 \\ 0 & |C_2|^2 & C_1^* C_2 & 0 \\ 0 & C_1 C_2^* & |C_1|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{13}$$

Now, we will calculate the entanglement of the atoms. Recently, different entanglement criteria for the optical systems has been proposed in [17, 18]. Here, we use the concurrence to evaluate the entanglement proposed in [17]. The concurrence of a state is defined as

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{14}$$

where $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ are the eigenvalues of the Hermitian matrix $R \equiv \sqrt{\sqrt{\rho}\widetilde{\rho}\sqrt{\rho}}$. Alternatively, it can be thought that λ_i is the square root of the eigenvalue of the matrix $\rho\widetilde{\rho}$. For a general state ρ of two qubits, the spin-flipped state is

$$\widetilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \tag{15}$$

where σ_y is the usual Pauli matrix, and ρ^* denotes the complex conjugation of the matrix.

After some calculations, the result of concurrence is listed in the following

$$C_a = 2|C_1| * |C_2|, \tag{16}$$

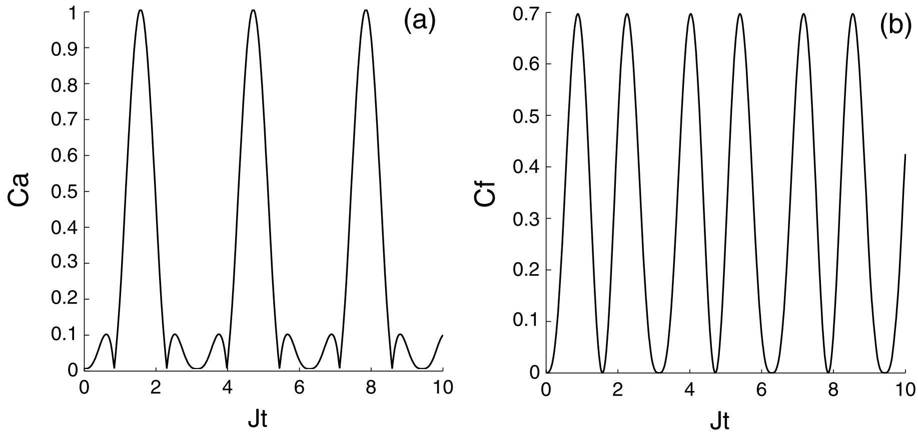

Fig. 3 (**a**) The concurrence of atomic entangled state as a function of time Jt. (**b**) The concurrence of field entangled state as a function of time Jt. The parameters are both $g_1 = g_2 = 5$, $\Omega_1 = \Omega_2 = 5$, $\Delta_1 = \Delta_2 = 100$, $N = 60$

Figure 3(a) shows the time evolution of the concurrence, which measures the entanglement of the atoms. We see that under this group of parameters, the concurrence reaches its maximum value 1 periodically. Through analyzing the results of (11), it is easy to see that the atomic state evolves into the maximally entangled state $|\Psi\rangle = \pm\frac{1}{\sqrt{2}}(|1,a\rangle|0,b\rangle - |0,b\rangle|1,a\rangle)$, when the two conditions $Jt = \frac{k\pi}{2}, k = 1, 3, 5, \ldots; \frac{at}{2} = k'\pi, k' = 1, 2, 3, \ldots$ are satisfied. It can be brought to a conclusion that we are able to generate the ideal atomic entangled state.

In the similar way, we trace over the part of atoms and write the density matrix ρ_f as follows

$$\rho_f = \mathrm{Tr}_a|\Psi\rangle\langle\Psi| = \begin{pmatrix} |C_1|^2 + |C_2|^2 & 0 & 0 & 0 \\ 0 & |C_4|^2 & C_3^* C_4 & 0 \\ 0 & C_3 C_4^* & |C_3|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{17}$$

After some calculations, the result of concurrence is listed in the following

$$C_f = 2|C_3| * |C_4|, \tag{18}$$

As shown in Fig. 3(b), the concurrence can not reach its maximum value 1. From (11), it is easily seen that we can not obtain the maximally field entangled state.

5 Effects of Photon Leakage

First, we will study the influence of photon leakage out of the cavities on the quantum state transfer. The master equation of motion for the density matrix of the entire system can be written as

$$\dot{\rho} = -i[H, \rho] + \kappa \sum_{j=1}^{2} (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j), \tag{19}$$

where κ is the rate for photon leakage out of the cavities. We consider the state transfer $(\alpha|0,b\rangle_1 + \beta|1,a\rangle_1) \otimes |0,b\rangle_2 \rightarrow |0,b\rangle_1 \otimes (\alpha|0,b\rangle_2 + \beta|1,a\rangle_2)$. In this case, the master equation can be numerically solved. In Fig. 4, we plot the probability of the state transfer as a function of Jt which first increases to the maximal value 0.9, and then drops down to zero. We find that the photon leakage out of the cavities has a deep influence on transfer efficiency. Therefore, a high-Q cavity is preferred. Next step, we calculate the atomic concurrence. As shown in Fig. 5, when we choose the specific parameters, we find that the concurrence oscillates periodically with time which is the same as the situation of no photon leakage. By comparison, the maximal value of concurrence gradually decreases. The main reason is that there is photon leakage out of the cavities.

6 Conclusion

In this paper, we consider a system consisting of two coupled cavities and multiple three-level atoms are trapped in the cavities. We present a scheme for implementing the ideal quantum state transfer between the two subsystems. In addition, we find that under certain conditions the atoms between two cavities can be maximum entanglement while the two

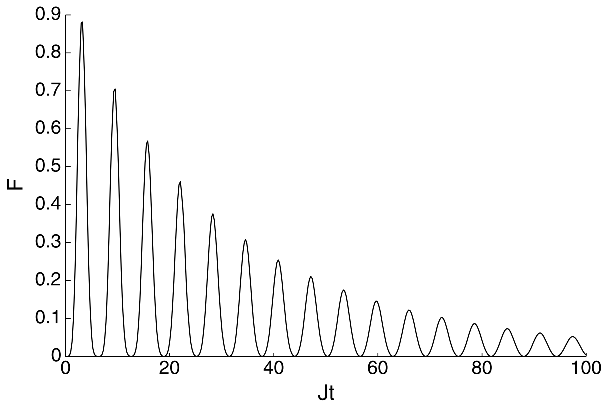

Fig. 4 The probability of the state transfer considering the photon leakage out of the cavities as a function of time Jt. $\kappa = 0.05$, $g_1 = g_2 = 1$, $\Omega_1 = \Omega_2 = 2$, $\Delta_1 = \Delta_2 = 20$, $N = 75$

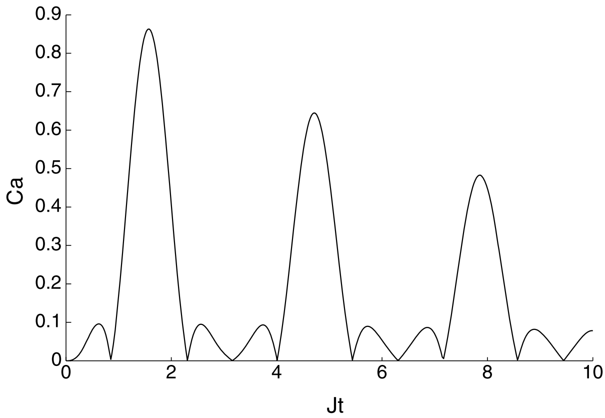

Fig. 5 The atomic concurrence considering the photon leakage as a function of time Jt. $\kappa = 0.1$, $g_1 = g_2 = 5$, $\Omega_1 = \Omega_2 = 5$, $\Delta_1 = \Delta_2 = 100$, $N = 60$

cavity fields can not achieve maximum entangled state. The effects of cavity losses on the probability of the state transfer as well as on the atomic entangled state are also investigated.

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